Krbcová and Kukal *EURASIP Journal on Image and Video Processing* (2017) 2017:83
DOI 10.1186/s13640-017-0232-z

EURASIP Journal on Image and Video Processing

RESEARCH Open Access

Relationship between entropy and SNR changes in image enhancement

Zuzana Krbcova[1*] and Jaromir Kukal[1,2]

Abstract

There are many techniques of image enhancement. Their parameters are traditionally tuned by maximization of SNR criterion, which is unfortunately based on the knowledge of an ideal image. Our approach is based on Hartley entropy, its estimation, and differentiation. Resulting gradient of entropy is estimated without knowledge of ideal images, and it is a subject of minimization. Both SNR maximization and gradient magnitude minimization cause various settings of the given filter. The optimum settings are compared, and their differences are discussed.

Keywords: Entropy, Signal-to-noise ratio, Robust signal-to-noise ratio, Filter design optimization

1 Introduction

In many different fields, image quality measurement is important for various image processing tasks. Traditional tasks as image enhancement [1, 2], sharpening, and smoothing are solved by digital filters of various types and parameter settings. Filter performance can be compared by different image quality assessment techniques [3, 4]. Image quality measure signal-to-noise ratio (SNR) [5] or its modifications are the most commonly used to compare filter performance [6–8]. SNR measure is based on the knowledge of referential image which is a kind of Full-Reference Image Quality Assessment. However, the original image is not available in real-world tasks. Therefore, No-Reference Image Quality Assessment (NR-IQA) technique [9, 10] must be used to measure image quality. Our approach is focused on relationship between SNR and Hartley entropy. In this paper, a novel NR-IQA method based on image entropy is introduced and verified on image dataset. Alternative approach focused on motion estimation and parallel computing is included in [11, 12].

2 Methods

2.1 Quality measures

A digital image is a 2D discrete signal obtained by a sampling process of analogous 2D signal. A digital image will be denoted by real function $\mathrm{x}(n_1, n_2)$ which describes image amplitude at an integer coordinate position (n_1, n_2). Image quality can be measured by standard measures as mean squared error or SNR. However, both mentioned measures are based on the knowledge of original image. Other measures must be used when original image is not known. Founding a relationship between SNR and entropy allows us to use also entropy as image quality measure.

2.1.1 Signal-to-noise ratio

The SNR is an image property comparing the ratio of signal power to noise power. The SNR measure can be used to analyze image quality. The estimation of SNR is based on knowledge of original undegraded image $\mathrm{s}(n_1, n_2)$. The SNR of input noisy image $\mathrm{x}(n_1, n_2)$ is calculated in the spatial domain as

$$SNR_x = 10 \log_{10} \frac{\mathrm{E}\left[\mathrm{s}(n_1, n_2)^2\right]}{\mathrm{E}\left[(\mathrm{x}(n_1, n_2) - \mathrm{s}(n_1, n_2))^2\right]} \tag{1}$$

with $\mathrm{E}[\cdot]$ standing for an expected value. The SNR of improved image $\mathrm{y}(n_1, n_2)$ is

$$SNR_y = 10 \log_{10} \frac{\mathrm{E}\left[\mathrm{s}(n_1, n_2)^2\right]}{\mathrm{E}\left[\left(\mathrm{y}(n_1, n_2) - \mathrm{s}(n_1, n_2)\right)^2\right]}. \tag{2}$$

The traditional improving measure ΔSNR is defined as a difference between SNR_y and SNR_x, and it allows to compare filter performances.

$$\Delta SNR = 10 \log_{10} \frac{\mathrm{E}\left[(\mathrm{x}(n_1, n_2) - \mathrm{s}(n_1, n_2))^2\right]}{\mathrm{E}\left[\left(\mathrm{y}(n_1, n_2) - \mathrm{s}(n_1, n_2)\right)^2\right]}. \tag{3}$$

*Correspondence: zuzana.krbcova@vscht.cz
[1] Department of Computing and Control Engineering, University of Chemistry and Technology, Prague, Technická 5, 166 28 Prague, Czech Republic
Full list of author information is available at the end of the article

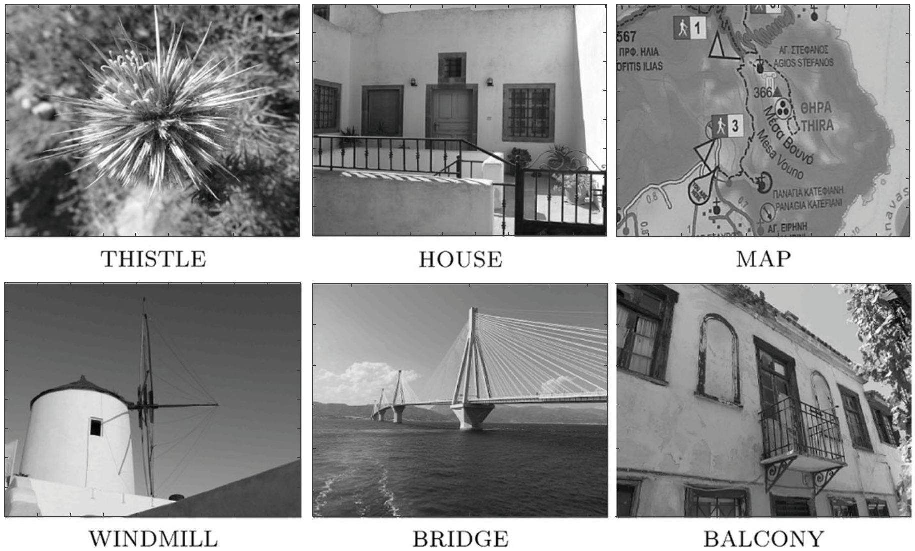

Fig. 1 Portfolio of original images

Positive ΔSNR value expresses the improvement of noisy image after its reconstruction. On the other hand, negative value expresses noisy image degradation.

2.1.2 *Robust signal-to-noise ratio*

Measure ΔSNR compares squared error of image intensities. Image filtering can cause intensities shifting or scaling, which will automatically decrease image quality measure ΔSNR. Therefore, we introduce robust version of ΔSNR designated as ΔR. Definition of ΔR is similar to ΔSNR but ΔR compares squared errors of statical rank

$$\Delta R = 10 \log_{10} \frac{\mathrm{E}\left[(\mathrm{R}(\mathrm{x}(n_1, n_2)) - \mathrm{R}(\mathrm{s}(n_1, n_2)))^2\right]}{\mathrm{E}\left[\left(\mathrm{R}(\mathrm{y}(n_1, n_2)) - \mathrm{R}(\mathrm{s}(n_1, n_2))\right)^2\right]}, \quad (4)$$

where $\mathrm{R}(\cdot)$ is a rank function [13] returning the rank of a pixel intensity inside an image. This measure is shift and scale invariant, but its time complexity is greater than time complexity of ΔSNR due to embedded sorting.

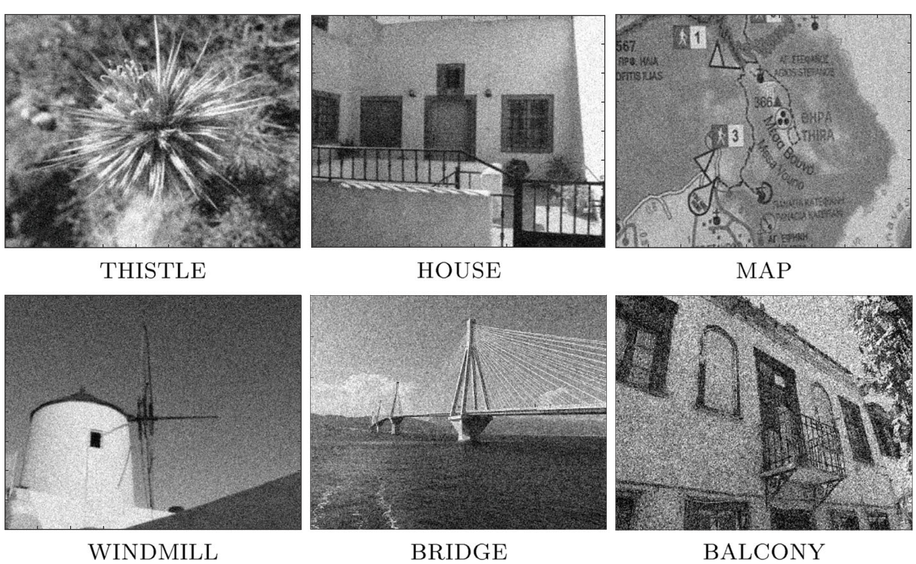

Fig. 2 Input images after degradation

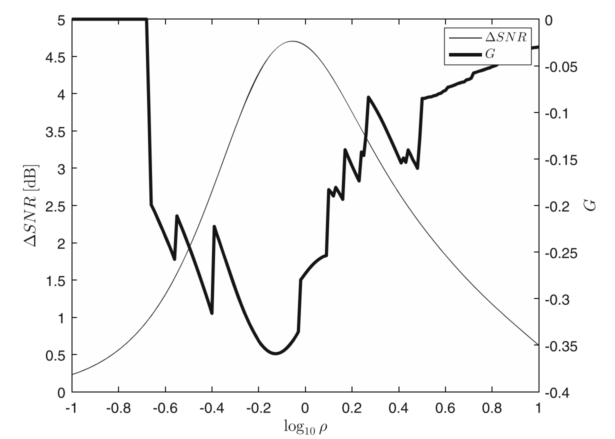

Fig. 3 Quality of Φ_1 sharpening as ΔSNR and G for MAP image

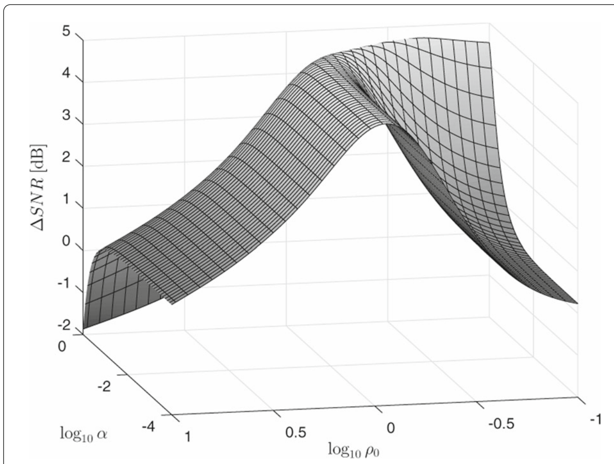

Fig. 5 Quality of Φ_3 sharpening with $\rho_1 = 0.9, \rho_2 = 0.2$ as ΔSNR for MAP image

2.1.3 Hartley entropy

Entropy is well known as a measure in statistical thermodynamics and information theory. We use entropy as a measure for image quality. To estimate image entropy, we use entropy estimation algorithm described in [14, 15]. Let $n \in \mathbb{N}$ be the number of image pixels, $x_k \in [0, 1]$ be the intensity of kth pixel for $k = 1, \cdots, n$, and $\varepsilon \in (0, 0.5]$ be the width parameter. Hartley entropy [16] (in nats) can be estimated as

$$\hat{H}(\varepsilon) = \ln \frac{\mu(\mathcal{C})}{2\varepsilon}, \tag{5}$$

where $\mu(\mathcal{C})$ is a measure of a set

$$\mathcal{C} = \left(\bigcup_{k=1}^{n} (x_k - \varepsilon, x_k + \varepsilon) \right) \cap (0, 1). \tag{6}$$

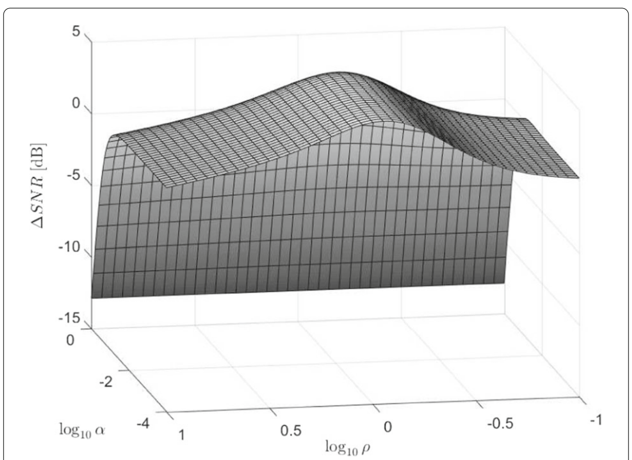

Fig. 4 Quality of Φ_2 sharpening as ΔSNR for MAP image

The measure of a set $\mu(\mathcal{C})$ can be calculated as

$$\mu(\mathcal{C}) = x_{(1)} + x_{(n)} - 1 + \sum_{k=1}^{n-1} x_{(k+1)} - x_{(k)} - 2\varepsilon, \tag{7}$$

where $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$. Supposing, a reconstructed image is the result of any filter application with parameters $\mathbf{p} \in \left(\mathbb{R}_0^+\right)^q$ where $q \in \mathbb{N}$ is a number of parameters.

The novel characteristic which helps to optimize digital filter design is the component wise maximum of Hartley entropy gradient

$$G = \max_{i=1,\cdots,q} \frac{\partial \hat{H}(\mathbf{p})}{\partial p_i} \tag{8}$$

that should be minimum possible which is the main supposition and matter of novel approach. The G criterion design is motivated as follows. When the filter has only one parameter ($q = 1$), we minimize $\partial \hat{H}/\partial p < 0$. Therefore, we obtain inflection point of $\hat{H}(p_1)$ for value of p_1 where the Hartley entropy rapidly decreases. The generalization for $q \in \mathbb{N}$ is based on minimax approach when we minimize the maximal parameter sensitivity $\partial \hat{H}/\partial p_i < 0$ over all tuning parameters. Whenever any $\partial \hat{H}/\partial p_i \ge 0$,

Table 1 Optimal low-pass smoothing Φ_1 via ΔSNR maximization

Image	Quality measures		Parameter
	ΔSNR	ΔR	$\log_{10} \rho$
THISTLE	3.897	3.990	−0.059
HOUSE	3.308	3.604	0.131
MAP	4.706	3.324	−0.053
WINDMILL	9.014	7.632	0.184
BRIDGE	4.578	3.839	−0.140
BALCONY	5.916	4.398	−0.032

Table 2 Optimal sharpening Φ_2 via ΔSNR maximization

Image	Quality measures		Parameters	
	ΔSNR	ΔR	$\log_{10}\alpha$	$\log_{10}\rho$
THISTLE	3.881	3.968	−2.203	−0.074
HOUSE	3.284	3.582	−2.159	0.102
MAP	4.705	3.306	−2.520	−0.062
WINDMILL	8.818	7.054	−2.124	0.089
BRIDGE	3.943	3.603	−1.417	−0.238
BALCONY	5.835	4.361	−1.802	−0.054

we set $G = 0$. The partial derivative of Hartley entropy $\hat{H}(\mathbf{p})$ with respect to the variable p_i can be approximated by finite differences

$$\frac{\partial \hat{H}(\mathbf{p})}{\partial p_i} \approx \frac{\hat{H}(\ldots, p_i + h, \ldots) - \hat{H}(\ldots, p_i - h, \ldots)}{2h}, \quad (9)$$

where spacing $h > 0$ approaches zero and $i \in \{1, \cdots, q\}$.

2.2 Linear filter primer

We have to introduce sharpening filters that will be used for studying relationship between SNR and entropy changes in image enhancement. Our interest [15] is focused only on linear infinite impulse response (IIR) filters [17] with radial symmetry in frequency domain whose response can be easily calculated by the Discrete Fourier Transform [18] (DFT). Their advantage is in the side effect suppression of a rectangular grid.

Let $\omega = \|\boldsymbol{\omega}\|_2, \boldsymbol{\omega} \in \mathbb{R}^2$ be the angular frequency and $\rho > 0$ be the radius. The radial filter has transfer function $\mathrm{F}(\boldsymbol{\omega}) = \Phi(\boldsymbol{\omega})$. Useful low-pass (LP) filter is a Gaussian filter [19] as traditional one

$$\Phi_1(\boldsymbol{\omega}) = \exp(-\rho^2\omega^2/2). \quad (10)$$

The simplest sharpening filter based on Gaussian filter with sharpening parameter $\alpha > 0$ and its generalization include

$$\Phi_2(\boldsymbol{\omega}) = \mathrm{LP}(\boldsymbol{\omega}) + \alpha(1 - \mathrm{LP}(\boldsymbol{\omega})), \quad (11)$$

$$\Phi_3(\boldsymbol{\omega}) = \mathrm{LP}_0(\boldsymbol{\omega}) + \alpha(\mathrm{LP}_1(\boldsymbol{\omega}) - \mathrm{LP}_2(\boldsymbol{\omega})), \quad (12)$$

where $\mathrm{LP}(\boldsymbol{\omega})$, $\mathrm{LP}_0(\boldsymbol{\omega})$, $\mathrm{LP}_1(\boldsymbol{\omega})$, and $\mathrm{LP}_2(\boldsymbol{\omega})$ are four realizations of low-pass filter $\Phi_1(\boldsymbol{\omega})$. In the case of Φ_2, only fundamental low-pass filter is used, but in Φ_3, the difference between two low-pass filters $(\mathrm{LP}_1, \mathrm{LP}_2)$ is used as high-pass filter added to the fundamental LP_0 filter in accordance with conventions of image processing. The filters Φ_1 (smoothing), Φ_2, and Φ_3 (sharpening) will be subject of parameter optimization in the next section. The filter Φ_1 has only one parameter ρ which is an advantage for its optimization. The filters Φ_2 and Φ_3 have two (α, ρ) and four $(\alpha, \rho_0, \rho_1, \rho_2)$ parameters, respectively. Their tuning can be performed by any heuristics for multimodal function optimization. Both Φ_1 and Φ_2 quality measures $(\Delta SNR, G)$ can be easily visualized.

3 Results and discussion

The novel characteristic G was tested on real images with an additive noise. The role of filter parameters was investigated for $\log_{10}\rho \in [-1, 1]$, $\log_{10}\alpha \in [-4, 0]$, and $\log_{10}\rho_k \in [-1, 0]$ where $k \in \{0, 1, 2\}$ in the case of Φ_1, Φ_2, and Φ_3. The Fast Simulated Annealing (FSA) [20] was used for ΔSNR maximization and G minimization inside given logarithmic ranges.

3.1 Test data

Four gray scale images (THISTLE, HOUSE, MAP, WINDMILL) of size 450×400 and two gray scale images (BRIDGE, BALCONY) of size 375×282 pixels were chosen to demonstrate the relationship between ΔSNR and G criteria. All image intensities were transformed from their original range to the interval $[0, 1]$ and were degraded by a box filter with squared mask of size 3×3 and then by Gaussian additive noise with $\sigma = 0.01$. The original images and results of their degradation are depicted in Figs. 1 and 2.

3.2 Image enhancement based on ΔSNR

The ΔSNR criterion was used for the optimization of filters Φ_1, Φ_2, and Φ_3 as a reference. The dependency of ΔSNR on ρ is demonstrated on Fig. 3 for smoother Φ_1 and MAP image. The dependency of ΔSNR on α and ρ is depicted in Fig. 4 for sharpening filter Φ_2 and the same image. Similarly, Fig. 5 is showing the dependency of ΔSNR on α and ρ_0 of filter Φ_3 with $\rho_1 = 0.9$ and $\rho_2 = 0.2$. The numerical results of heuristics maximization are included in Tables 1, 2, and 3 for both traditional and referential approaches. Reconstructed images via Φ_3 with maximal ΔSNR are shown in Fig. 6.

Table 3 Optimal sharpening Φ_3 via ΔSNR maximization

Image	Quality measures		Parameters			
	ΔSNR	ΔR	$\log_{10}\alpha$	$\log_{10}\rho_0$	$\log_{10}\rho_1$	$\log_{10}\rho_2$
THISTLE	3.935	4.035	−0.137	−0.099	−0.217	−0.313
HOUSE	3.273	3.501	−0.077	−0.005	−0.100	−0.161
MAP	4.739	3.343	−0.148	−0.167	−0.079	−0.250
WINDMILL	8.706	6.833	−0.075	−0.012	−0.037	−0.162
BRIDGE	4.811	4.100	−0.230	−0.001	−0.491	−0.223
BALCONY	6.029	4.484	−0.621	−0.012	−0.481	−0.216

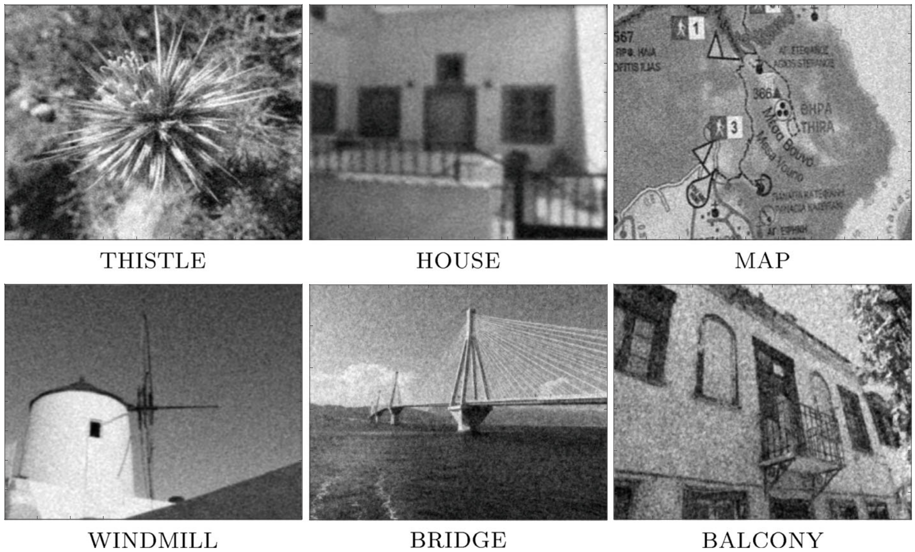

Fig. 6 Sharpened images via Φ_3 with maximum ΔSNR

3.3 Image enhancement based on *G*

The novel G measure was used for the optimization of filters mentioned above. The measure G was approximated by Eq. (9) with spacing $h = 10^{-12}$. Width parameter ε was set to value 0.01. The dependency of G on ρ is demonstrated on Fig. 3 for smoother Φ_1 and MAP image. The dependency of G on α and ρ is depicted in Fig. 7 for sharpening filter Φ_2 and the same image. For the last filter Φ_3 with $\rho_1 = 0.9$ and $\rho_2 = 0.2$, the dependency of G on α and ρ_0 is depicted in Fig. 8. The numerical results of heuristic minimization via FSA are included in Tables 4, 5, and 6 with adequate values of ΔSNR and ΔR. Reconstructed images via Φ_3 with minimal G are shown in Fig. 9.

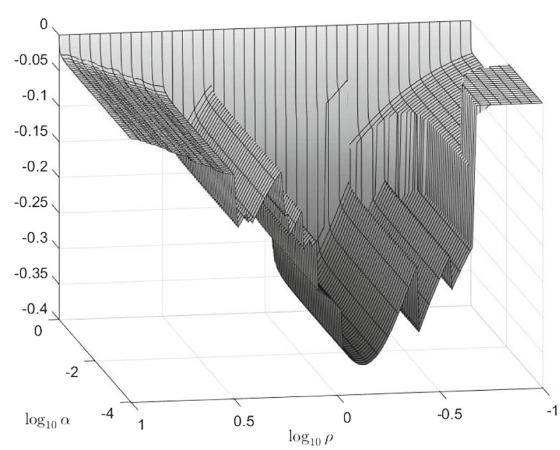

Fig. 7 Quality of Φ_2 sharpening as G for MAP image

3.4 Discussion

The proposed novel criterion G was minimized to obtain the optimal parameters of the three different filters tested on the real images. The quality of the optimal reconstruction was evaluated by the classical ΔSNR measure and our robust version ΔR. For a comparison, the same images were reconstructed by the filters whose optimal parameters were obtained by maximization of ΔSNR. The relative changes RC between the qualities of the optimal reconstruction according to the filters Φ_1, Φ_2, and Φ_3 evaluated for the criterion ΔSNR and G are summarized in the Table 7. When comparing the results, it can be seen that the achieved results are similar. The most considerable changes in the quality measures were obtained for the filter Φ_2 settings providing significantly lower qualities but still improving image enhancement. The image intensities reconstructed by the filter Φ_2 and proposed criterion G are shifted or scaled which results from the large values of the relative changes with respect to the quality measure ΔSNR.

4 Conclusions

The novel No-Reference Image Quality Assessment method and adequate criterion were introduced in this paper. It is based on the Hartley entropy estimation from gray-level densities and the optimization of its changes during tuning of filter parameters. Three types of linear image filters with various number of parameters were optimized by using traditional SNR criterion as a reference, first. Using novel criterion G and its minimization, similar results of comparable SNR quality were obtained without prior knowledge of ideal image. The novel procedure is directly applicable to real image enhancement.

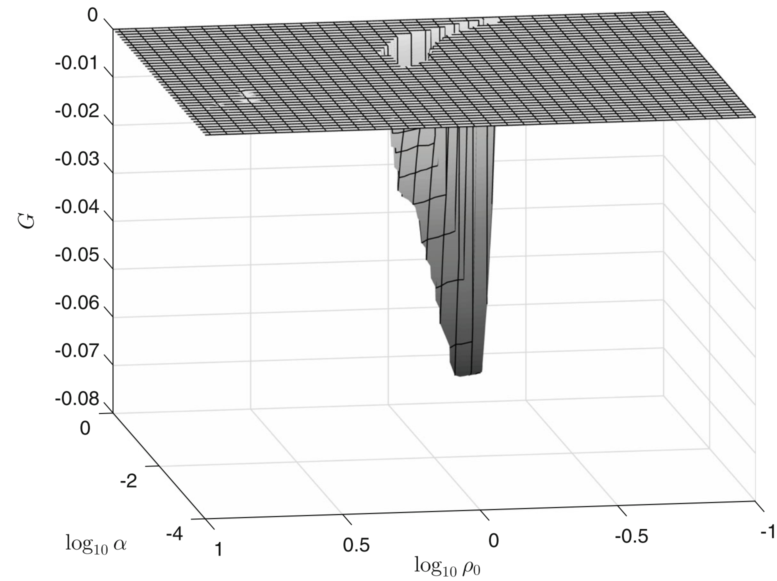

Fig. 8 Quality of Φ_3 sharpening with $\rho_1 = 0.9, \rho_2 = 0.2$ as G for MAP image

Table 4 Optimal low-pass smoothing Φ_1 via G minimization

Image	Quality measures			Parameter
	G	ΔSNR	ΔR	$\log_{10} \rho$
THISTLE	−0.126	3.862	3.898	−0.105
HOUSE	−0.140	3.197	3.377	−0.020
MAP	−0.360	4.613	3.133	−0.122
WINDMILL	−0.106	8.471	6.464	0.021
BRIDGE	−0.265	4.341	3.584	−0.242
BALCONY	−0.308	5.764	4.159	−0.120

Table 5 Optimal sharpening Φ_2 via G minimization

Image	Quality measures			Parameters	
	G	ΔSNR	ΔR	$\log_{10} \alpha$	$\log_{10} \rho$
THISTLE	−0.126	2.563	3.882	−1.121	−0.111
HOUSE	−0.408	−1.710	1.275	−0.652	−0.472
MAP	−0.358	3.841	3.145	−1.203	−0.119
WINDMILL	−0.154	−5.113	3.567	−0.503	−0.237
BRIDGE	−0.259	−9.055	3.580	−0.308	−0.243
BALCONY	−0.315	5.405	4.197	−1.343	−0.111

Table 6 Optimal sharpening Φ_3 via G minimization

Image	Quality measures			Parameters			
	G	ΔSNR	ΔR	$\log_{10} \alpha$	$\log_{10} \rho_0$	$\log_{10} \rho_1$	$\log_{10} \rho_2$
THISTLE	−0.025	3.869	3.900	−0.368	−0.284	−0.055	−0.895
HOUSE	−0.005	3.274	3.502	−0.348	−0.027	−0.006	−0.168
MAP	−0.077	4.608	3.199	−0.212	−0.310	−0.042	−0.666
WINDMILL	−0.008	7.309	5.130	−0.065	−0.257	−0.176	−0.412
BRIDGE	−0.005	3.596	2.859	−0.080	−0.329	−0.323	−0.927
BALCONY	−0.044	5.728	4.164	−0.196	−0.390	−0.009	−0.630

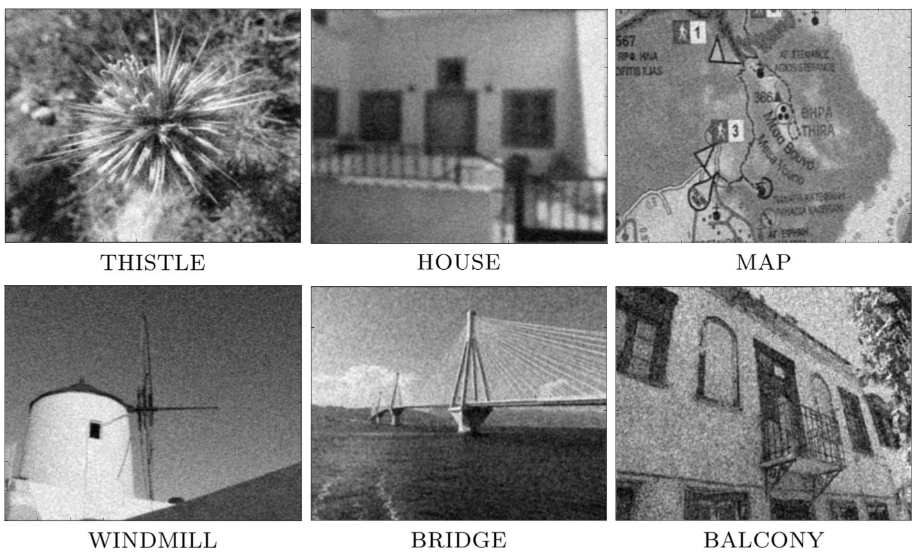

Fig. 9 Sharpened images via Φ_3 with minimum G

Table 7 The relative changes [%] between quality measures

Image	Φ_1		Φ_2		Φ_3	
	$RC_{\Delta \mathrm{SNR}}$	$RC_{\Delta R}$	$RC_{\Delta SNR}$	$RC_{\Delta R}$	$RC_{\Delta SNR}$	$RC_{\Delta R}$
THISTLE	0.90	0.03	33.96	2.17	1.68	3.35
HOUSE	3.36	6.30	152.07	64.41	0.03	0.03
MAP	1.98	5.75	18.36	4.87	2.76	4.31
WINDMILL	6.02	15.30	157.98	49.43	16.05	24.92
BRIDGE	5.18	6.64	329.65	0.64	25.25	30.27
BALCONY	2.57	5.43	7.37	3.76	4.99	7.14
Mean	3.33	6.58	116.57	20.88	8.46	11.67

Acknowledgements
Not applicable.

Funding
This study has been supported financially from specific university research MSMT no. 20-SVV/2016, CTU grant SGS 17/196/OHK4/3T/14 and grant GA17-05840S "Multicriteria Optimization of Shift-Variant Imaging System Models" of the Czech Science Foundation.

Availability of data and materials
The dataset supporting the conclusions of this article is included within the article.

Authors' contributions
JK suggested the main ideas of the research and realized a part of algorithms. ZK realized a part of algorithms and performed the comparison between entropy and SNR. ZK and JK took part in writing and approved the final version of the manuscript.

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Publisher's Note

Author details
[1]Department of Computing and Control Engineering, University of Chemistry and Technology, Prague, Technická 5, 166 28 Prague, Czech Republic.
[2]Department of Software Engineering, Czech Technical University, Prague, Trojanova 13, 120 00 Prague, Czech Republic.

Received: 14 June 2017 Accepted: 20 November 2017
Published online: 19 December 2017

References

1. B Langari, S Vaseghi, A Prochazka, B Vaziri, FT Aria, Edge-guided image gap interpolation using multi-scale transformation. IEEE Trans. Image Proc. **25**(9), 4394–4405 (2016)
2. E Hostalkova, O Vysata, A Prochazka, in *15th International Conference on Digital Signal Processing Location*. Multi-dimensional biomedical image de-noising using Haar transform (Cardiff Univ, Cardiff, WALES, 2007)
3. S Patil, S Sheelvant, Survey on image quality assessment techniques. Int. J. Sci. Res. **4**(7), 1756–1759 (2015)
4. P Hanhart, MV Bernardo, M Pereira, AMG Pinheiro, T Ebrahimi, Benchmarking of objective quality metrics for HDR image quality assessment. EURASIP J. Image Video Process. **2015**, 39 (2015)
5. RG Lyons, *Understanding digital signal processing*. (Prentice Hall, New Jersey, 2010)
6. I Djurović, BM3D filter in salt-and-pepper noise removal. EURASIP J. Image Video Process. **2016**, 13 (2016)
7. D-H Jiang, X Tan, Y-Q Liang, S Fang, A new nonlocal variational bi-regularized image restoration model via split Bregman method. EURASIP J. Image Video Process. **2015**, 15 (2015)
8. A Isar, S Moga, D Isar, A new denoising system for SONAR images. EURASIP J. Image Video Process. **2009**, 173841 (2009)
9. V Kamble, KM Bhurchandi, No-reference image quality assessment algorithms: a survey. Optik - Int. J. Light Electron. Opt. **126**(11-12), 1090–1097 (2015)
10. H Liu, I Heynderickx, in *Proc. SPIE 7867, Image Quality and System Performance VIII, vol. 78670C*. Issues in the design of a no-reference metric for perceived blur, (2011)
11. C Yan, et al, A highly parallel framework for HEVC coding unit partitioning tree decision on many-core processors. IEEE Sig. Process. Lett. **21**(5), 573–576 (2014)
12. C Yan, et al, Efficient parallel framework for HEVC motion estimation on many-core processors. IEEE Trans. Circ. Syst. Video Technol. **24**(12), 2077–2089 (2014)
13. EL Lehmann, HJM D'Abrera, *Nonparametrics, statistical methods based on ranks*. (HOLDEN-DAY, San Francisco, 1975)
14. Z Krbcová, J Kukal, in *MENDEL 2016 - 22nd International Conference on Soft Computing*. Image quality measures based on entropy (Brno: University of Technology, 2016), pp. 209–216
15. Z Krbcová, J Kukal, J Švihlík, K Fliegel, in *Proceedings of SPIE Volume 9971 Applications of Digital Image Processing XXXIX, vol. 997121*. Fast estimate of Hartley entropy in image sharpening, (2016)
16. RVL Hartley, Transmission of information. Bell Syst. Tech. J. **7**(3), 535–563 (1928)
17. JS Lim, *Two-dimensional signal and image processing*, 1st edn. (Prentice Hall PTR, New Jersey, 1990)
18. N Ahmed, T Natarajan, KR Rao, Discrete cosine transform. IEEE Trans. Comput. **C-23**, 90–93 (1974)
19. HJ Blinchikoff, AI Zverev, *Filtering in the time and frequency domains*. (Wiley, New York, 1976)
20. H Szu, R Hartley, Fast simulated annealing. Phys. Lett. A. **122**, 157–162 (1987)