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Research Article

Are the Wavelet Transforms the Best Filter Banks for Image Compression?

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Maximum regular wavelet filter banks have received much attention in the literature, and it is a general conception that they enjoy some type of optimality for image coding purposes. To investigate this claim, this article focuses on one particular *biorthogonal* wavelet filter bank, namely, the 2-channel 9/7. As a comparison, we generate all possible 9/7 filter banks with perfect reconstruction and linear phase while having a different number of zeros at $z = -1$ for both analysis and synthesis lowpass filters. The best performance is obtained when the filter bank has 2/2 zeros at $z = -1$ for the analysis and synthesis lowpass filters, respectively. The competing wavelet 9/7 filter bank, which has 4/4 zeros at $z = -1$, is thus judged inferior both in terms of objective error measurements and informal visual inspections. It is further shown that the 9/7 wavelet filter bank can be obtained using gain-optimized 9/7 filter bank.

1. INTRODUCTION

The transform is one of three major building blocks in waveform image compression systems, where quantization and coding are the two other blocks. It has been stated in the literature by many researchers that choice of decomposition transformation is a critical issue, which affects the performances of the image compression system.

There are some differences in designing filters in filter banks compared with wavelet transforms. Wavelet filters are designed using associated continuous scaling functions and iterations. The filters in filter banks do not have to be associated with a single filter or basis function. They can be designed and optimized in many ways. However, the most commonly used image compression systems employ filters with perfect reconstruction (PR), finite impulse response (FIR), and linear phase, and they are nonunitary (biorthogonal). It should be noted that when more constraints are imposed on a filter bank, fewer variables will be available for optimization.

Appropriate filter design criteria adapted to our visual perception used for image compression still remain an unsolved issue. For wavelet filters it has been proposed to have biorthogonal, maximum regularity, minimum shift-variance, minimum impulse response peak to sidelobe peak ratio, step response ratio, and so on [1, 2]. The filter bank designers on the other hand have proposed relaxation of perfect reconstruction, shorter synthesis highpass/bandpass filters, maximum coding gain, "bell-shape" synthesis lowpass filter, half-whitening property in analysis lowpass filter, and so on [3–9].

The ideal frequency separation between bands is, from an implementation point of view, impossible. Furthermore, subjectively it is also not a good idea. One type of problem resulting from long impulse responses (this is the consequence of filters with ideal frequency separation) is the so-called ringing artifact. This is related to Gibb's phenomenon. Assume that the signal is to be reconstructed from the lowpass band only because the signal level would be lower than the quantization noise level in all other bands. Then edges in the image would be rendered as edges plus damped "echoes" of the edges due to the strong variations of the tails in the impulse response in an ideal filter. In practice, one has to find a balance between the desirability of high gain and other subjectively important measures while using moderate length filters.

One of the objectives of this paper is to study 2-channel 9/7 *biorthogonal* filter banks. We derive all possible filter banks that have PR and linear phase properties and show that *biorthogonal* wavelet filters can be obtained by using appropriate number of zeros on the unit circle, where remaining degrees of freedom are used to maximize for subband coding gain. Furthermore, we show that optimal filters can be obtained by relaxing maximum regularity constraint used in the wavelet theory, where the additional degrees of freedom can be used for subband coding gain. Both the wavelet and gain optimized filters are compared in a JPEG 2000 compliant image compression scheme, where objective error measurements and subjective assessments will be given.

2. DECOMPOSITION TRANSFORMS

The transform is meant to transfer the signals from one domain into another, where signal dependencies (correlations) are removed. The quantization renders a digital representation of the signal parameters while allowing a certain signal degradation, while coding is used for efficient bit representation.

The design criteria used in the wavelet transforms and filter banks differ, and the rest of this section is devoted to this topic.

2.1. *Filter banks*

Two-channel uniform filter banks are considered in the following. We enforce PR in the following way, where $H_{\text{LP}}(z)$ is a lowpass (LP) filter, and $H_{\text{HP}}(z)$ is a highpass (HP) filter. The filters can be described in polyphase form as

$$\mathbf{H}(z) = \begin{bmatrix} H_{\text{LP}}(z) \\ H_{\text{HP}}(z) \end{bmatrix} = \begin{bmatrix} P_{00}(z^2) & P_{01}(z^2) \\ P_{10}(z^2) & P_{11}(z^2) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} = \mathbf{P}(z^2)\mathbf{d}(z), \quad (1)$$

where the polyphase matrix, $\mathbf{P}(z)$ and the delay vector, $\mathbf{d}(z)$, are easily identified in this equation [10].

Denoting the polyphase reconstruction filter matrix by $\mathbf{Q}(z)$, a sufficient condition for PR can be expressed as [11]

$$\mathbf{Q}(z) = z^{-k}\mathbf{P}^{-1}(z), \quad (2)$$

where k is an integer representing a necessary delay. Given FIR analysis filters, FIR synthesis filters are obtained by setting all coefficients except one to zero in the polynomial representing the determinant of $\mathbf{P}(z)$. Denoting the synthesis filters by $G_{\text{LP}}(z)$ and $G_{\text{HP}}(z)$, respectively, the above condition implies that $G_{\text{LP}}(z) = H_{\text{HP}}(-z)$ and $G_{\text{HP}}(z) = -H_{\text{LP}}(-z)$. Observe the close connection between the analysis and synthesis filters which simply represents an LP to HP transform through frequency shifts by π.

Table 1: Possible combinations that give zeros at $z = -1$.

Number of zeros	Solution	Gain (dB)	Number of zeros	Solution	Gain (dB)
0/0	yes	6.505	4/4	yes	5.916
0/2	yes	6.498	4/6	no	—
0/4	yes	6.319	6/0	yes	1.015
0/6	yes	3.371	6/2	yes	0.910
2/0	yes	6.505	6/4	no	—
2/2	yes	6.496	6/6	no	—
2/4	yes	6.266	8/0	yes	−30.123
2/6	yes	3.070	8/2	no	—
4/0	yes	6.505	8/4	no	—
4/2	yes	6.305	8/6	no	—

The above constraints are the most general to construct PR system having FIR filters. If linear phase filters are desired, the system becomes nonunitary (biorthogonal).

2.2. *Regularity constraint*

In wavelet theory, regularity has been defined as a smoothness measure of a wavelet transform. It has been shown that a wavelet to have regularity, the analysis and synthesis lowpass filters $H_{\text{LP}}(z)$ and $G_{\text{LP}}(z)$ should have a sufficient number of zeros at $z = -1$. Consequently, it can be stated that if $H_{\text{LP}}(z)$ has N zeros at $z = -1$, the corresponding synthesis highpass filter, $G_{\text{HP}}(z)$ will have N vanishing moments [12]. A study on maximum regularity in orthogonal systems can be found in [13]. However, our focus in this paper is only for *biorthogonal*, *linear phase* systems.

Let us investigate the importance of zeros at $z = -1$ for the analysis and synthesis lowpass filters. A hypothesis is that in order to alleviate perceptually annoying noise, the DC gain of the odd and even polyphase lowpass synthesis filter components should be equal. This will prevent the generation of a periodic output from the synthesis filter whenever the input is constant and will also reduce cyclostationary noise in general. This requirement will force at least one zero to be exactly at $z = -1$ for odd length lowpass filters.

Consider the synthesis lowpass filter written in polyphase form:

$$G_{\text{LP}}(z) = Q_{00}(z^2)z^{-1} + Q_{01}(z^2). \quad (3)$$

A zero at $z = -1$ is equivalent to

$$G_{\text{LP}}(-1) = -Q_{00}(1) + Q_{01}(1) = 0, \quad (4)$$

which implies that $Q_{00}(1) = Q_{01}(1)$. This is exactly the equality between the DC amplification of the two polyphase components.

Now for odd length, lowpass, *linear phase* FIR filters with one zero at $z = -1$, an additional zero would also have to be placed at the same position. Or in general, zeros at $z = -1$ must appear in pairs.

Table 2: Wavelet and gain optimized filters for 4/4 zeros $z = -1$.

Wavelet filters		Gain optimized filters	
H_{LP}	G_{LP}	H_{LP}	G_{LP}
0.03750420174433	−0.06509620731678	0.03741392086701	−0.06531200385798
−0.02364485850165	−0.04104029469797	−0.02375429115352	−0.041588241345452
−0.10967708612048	0.42170166115821	−0.1095444797283	0.42145506934004
0.37417153290464	0.79529351434610	0.37423343534046	0.79546261365501
0.84540010899851		0.84521940609657	

It should be noted that for even length filters there will always be at least one zero at $z = -1$. The DC gain condition can also be seen to be satisfied by observing that the coefficients of the two polyphase filters are reversed versions of each other.

Another feature which seems important is that as images have strong low-frequency components, the analysis highpass filter should have at least one zero at $z = 1$. But this is equivalent to the previous requirement due to the derived relationship between analysis and synthesis filters.

The question is now, do we get even better performance by increasing the multiplicity of these zeros?

To scrutinize this problem, we investigate a 9/7 filter bank.

2.3. 9/7 Perfect reconstruction linear phase transforms

The analysis 9/7 filter pairs can be written as

$$\begin{aligned} H_{LP}(z) &= 1 + a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3} + a_3 z^{-4} + a_2 z^{-5} \\ &\quad + a_1 z^{-6} + a_0 z^{-7} + z^{-8}, \\ H_{HP}(z) &= 1 + b_0 z^{-1} + b_1 z^{-2} + b_2 z^{-3} + b_1 z^{-4} + b_0 z^{-5} + z^{-6}. \end{aligned} \tag{5}$$

We assume using optimum bit allocation to quantize the analysis samples as described in [14]. Then we can write the subband coding gain relative to pulse code modulation (PCM) as

$$G_{SBC} = \frac{\sigma^2_{r_{PCM}}}{\sigma^2_{r_{opt}}} = \frac{1}{\prod_{i=0}^{1} (\mathbf{h}_i^T \mathbf{R}_{xx} \mathbf{h}_i \mathbf{g}_i^T \mathbf{g}_i)^{1/2}}. \tag{6}$$

Here $\mathbf{R}_{xx}$ is the autocorrelation matrix of the input signal $x(n)$ where the entries are $\mathbf{R}_{xx}(i, j) = E[x(i)x(j)]$, and $\mathbf{h}_i$ and $\mathbf{g}_i$ are the ith channel's analysis and synthesis filter vectors, respectively. Furthermore, $\sigma_r^2 = \sum_{i=0}^{1}(1/2)\mathbf{g}_i^T \mathbf{g}_i \sigma^2_{q_i}$, where $\sigma^2_{q_i}$ denotes quantization noise in the ith channel.

There are 20 possible combinations to have zeros at $z = -1$, as given in Table 1. (Number of zeros means: number of zeros at $z = -1$ for lowpass: analysis/synthesis filters.) However, as shown in the table, not all possible combinations of zeros at $z = -1$ will satisfy the PR and linear phase properties. This means we have only 14 combinations. In the case of 4/4 zeros at $z = -1$, the 9/7 wavelet [12] and gained optimized filter banks coincide, and are, in fact, the only possibility. The filter coefficients are given in Table 2. The rest of the filter coefficients can be found by using the symmetric property. Note that the synthesis filters have unit gain, that is, their l_2 norm is equal to 1, which implies that $\sigma_r^2 = (1/2)[\sigma^2_{q_1} + \sigma^2_{q_2}]$.

3. OPTIMIZATION STRATEGIES: SUBBAND CODING GAIN

After linear phase and PR being imposed on a filter bank, the remaining degrees of freedom can be used for gain optimization (see (6)), or more importantly, to achieve subjectively good performance. It is obvious that the more degrees of freedom that can be exploited towards a given optimization criterion, the better. The correspondence between subjective criteria and simple mathematical criteria, as used presently, is rather poor. Typically, filter banks are designed to minimize the mean square error (MSE) after signal decompression for a given source statistics and quantization scheme. Furthermore, encapsulating subjective performance criteria into a set of mathematical equations which can be incorporated into an overall optimization criterion is warranted.

We choose the cost function to be defined in terms of coding gain, which is given in (6). The coding gain can be seen as a measure to assess the data compression ratio [15]. Katto and Yasuda [4] generalized the measure to be used in biorthogonal, nonuniform (e.g., wavelet tree) filter banks.

In the literature, it has been argued that most natural images can be approximated as an autoregressive (AR) process, where the nearest sample autocorrelation coefficient $\rho = 0.95$. We will also use this model, implying that

$$\mathbf{R}_{xx} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{7}$$

will be used in (6). We used the "Optimization Toolbox" in Matlab to optimize the cost function.

Table 1 lists the coding gain optimization results for all possible configurations, of these the following have poor coding gain (increasing gain order): 8/0, 6/2, 6/0, 2/6, and 0/6. There remain 7 possible zero combinations with gains in the range 5.92 dB to 6.51 dB, where the 4/4 case (the wavelet case) is inferior to the others. To make a comparison with the wavelet transform, we rule out the 0/0, 2/0, and 4/0 cases, as these lack the necessary regularity constraint. The 0/2 and 2/2 choices seem to be the best among the remaining configurations. In peak-signal-to-noise ratio (PSNR) comparisons, the 2/2 case performed slightly better than 0/2 case [16]. Therefore, we choose the 2/2 configuration.

Figure 1 shows the frequency responses of the gain-optimized filter bank with 2/2 zeros at $z = -1$ and the wavelet filter bank. The passband of the analysis optimized lowpass filter is slightly elevated, which is referred to as the half-whitening property in [7, 15]. Only a crude

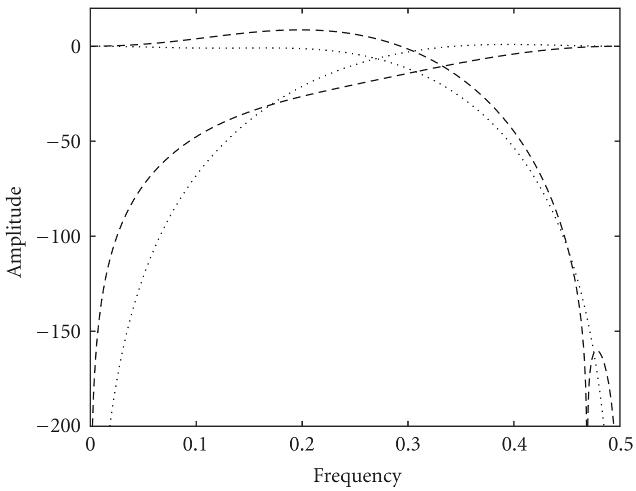


Figure 1: Frequncy response of the analysis filters. Gain optimized 2/2 zeros at $z = -1$ (dashed) and wavelet 4/4 zeros at $z = -1$ (dotted).

approximation to the half-whitening property of the signal spectrum can be obtained with short length FIR filters.

Table 3 lists the gain optimized 2/2 case of the 9/7 filter coefficients for 6 levels. Only the first 5 and 4 filter coefficients of the analysis lowpass (h_{LP}) and synthesis lowpass (g_{LP}) are listed, respectively. By using the symmetric and modulation properties, highpass filter coefficients can be found. The filter coefficients have different values in each level indicating that the power spectrum in each level is different.

In the case of 4/4 zeros at $z = -1$, the wavelet 9/7 filter bank [12] and gain optimized 9/7 filter bank have almost identical filter coefficients as given in Table 2. Their zero location diagrams are shown in Figure 2, whereas the zero location diagrams for 2/2 case of the 9/7 filter bank are shown in Figure 3.

Table 3: The gain optimized 9/7 filter bank (the 2/2 zeros at $z = -1$ case) analysis and synthesis lowpass filter coefficients.

$h_{LP}(1:5)^T$	$g_{LP}(1:4)^T$
Level-1	
−4.4985547417617e-02	7.6313567129747e-02
2.6332850768416e-02	4.4671097501108e-02
1.0569163480620e-01	−4.2815691469541e-01
−3.8160459560842e-01	−7.9302889013354e-01
−8.3195566445719e-01	
Level-2	
6.0365140306837e-02	−9.3024425657936e-02
−3.4502630852655e-02	−5.3169551208556e-02
−9.6343466902220e-02	−4.3174350950173e-01
4.0354000123490e-01	7.8377727010471e-01
8.1003139395526e-01	
Level-3	
5.5206559091025e-02	−9.1725776573471e-02
−2.7922407706617e-02	−4.6393120181020e-02
−1.0441747150424e-01	4.3633696256551e-01
3.9065004128636e-01	7.8200861234611e-01
8.2387709198593e-01	
Level-4	
−5.0936329872229e-02	8.8243072490171e-02
2.4607113245270e-02	4.2629833820440e-02
1.0875991200411e-01	−4.3726447799116e-01
−3.8275133600598e-01	−7.8330247864284e-01
−8.3193560978519e-01	
Level-5	
−4.0284391511432e-02	7.1303004882139e-02
2.3137486606188e-02	4.0953139877346e-02
1.0976902435909e-01	−4.2804933628869e-01
−3.7352651375374e-01	−7.9539894256780e-01
−8.3974731999042e-01	
Level-6	
−1.8269742086611e-02	3.2624711093588e-02
1.7911475762202e-02	3.1984946433911e-02
1.1753257439016e-01	−4.1036902770132e-01
−3.4882313876892e-01	−8.1945852608329e-01
−8.6034899062052e-01	

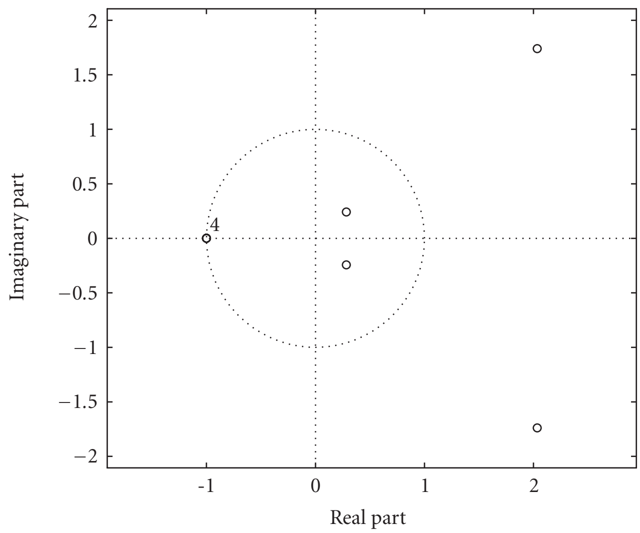
4. RESULTS

Gray scale test images such as *Bike*, *Cafe*, *Target*, and *Woman* were chosen from the JPEG 2000 test set (JPEG 2000 compression test image CDROM ISO/IEC JTC 1/SC 29/WG1) where a JPEG 2000 complaint image coder was employed in our experiment [17]. The bitrates used were 0.0625, 0.125, 0.25, and 0.5 bits/pixel (bpp). Furthermore, we have chosen to use the same objective error criteria used in the evaluation of the candidate image compression systems submitted to the JPEG 2000 comittee in 1997 in order to compare the competing filter banks, where only the peak-signal-to-noise ratio (PSNR) is presented in Table 4. The gain optimized filter bank performs better than the wavelet filter bank for image *Target*. For all other images the wavelet and gain optimized filter banks perform equally well. Comprehensive coding results for a number of filter banks and different frequency partitions can be found in [18, 19]. So the question now is whether the decoded images of both filter banks look the same.

During the evaluation of the JPEG 2000 candidates, an extensive subjective evaluation was performed. Both objective and subjective evaluations were used to select the system for further development. We do not have resources to perform a comprehensive subjective test. Let us rather inspect some images for annoying artifacts. If we compare the gain optimized 9/7 filter bank (2/2 zeros at $z = -1$) and the 9/7 wavelet filter bank (4/4 zeros at $z = -1$), the *ringing* artifact becomes severe in the 4/4 case. To explain this, we examine the synthesis lowpass filter's unit sample response. For simplicity, the unit sample response of a 3-level decomposition is shown in Figure 4. The unit sample responses of both 2/2 and 4/4 cases are obtained by convolving the unit sample responses of *each level*. For comparison purposes both filters are restricted to have unit l_2 norm. In Figure 4, we see that

Table 4: PSNR results: 9/7 wavelet and gain optimized filter banks.

Image	Filter bank					Wavelet				
	0.0625	0.125	0.25	0.5	Avg.	0.0625	0.125	0.25	0.50	Avg.
Bike	22.91	25.51	28.68	32.69	27.45	22.91	25.51	28.68	32.71	27.45
Cafe	19.00	20.63	23.05	26.53	22.30	19.02	20.66	23.08	26.58	22.34
Target	17.31	20.39	24.42	31.10	23.31	17.13	20.06	24.18	31.01	23.10
Woman	25.45	27.21	29.70	33.18	28.89	25.38	27.19	29.67	33.15	28.85



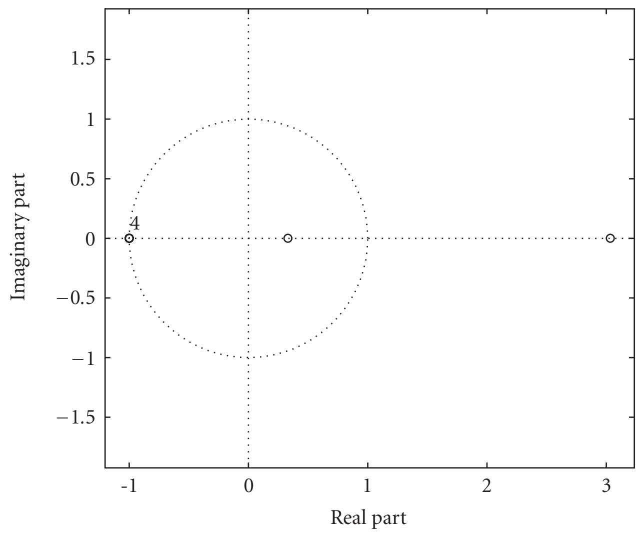


Figure 2: 4/4 zeros at $z = -1$ of the gain optimized and also wavelet 9/7 filter bank. (a) Analysis and (b) synthesis lowpass filters.

the magnitude of the side-lobes (negative unit sample values) of the 4/4 case is much larger than in the 2/2 case, and this leads to severe *ringing* at low-bit rates. Furthermore, severe *checker board* and *waveform* types of artifacts were observed for the cases of 0/0, 2/0, and 4/0 zeros at $z = -1$ [20]. The gain optimized 2/2 zeros at $z = -1$ had less ringing around sharp edges than the wavelet filter bank (see image *target* in Figure 5). Smooth regions and textures are better reconstructed by the gain optimized filter bank than the wavelet filter bank (see image *cafe* in Figure 6).

So far we have seen that the gain optimized and wavelet filter banks had similar objective measurements whereas there are some differences in their visual appearances. Let us see whether we can interpret our finding by inspecting the power spectra of the images. The calculated ρ in AR(1) model for the images, *Bike*, *Cafe*, *Target*, and *Woman*, are 0.97, 0.92, 0.76, and 0.97, respectively. Furthermore, *Woman* and *Target* have the larger power spectral variations. The larger the power spectral variations are, the higher the spectral flatness measure becomes [15]. The spectral flatness measure is used in the bit allocation scheme. This may be a reason that *Woman* and *Target* have slightly better PSNR measurements as given in Table 4.

The *Bike* and *Woman* images are best matched to the statistical model used in the optimization. For other images there is a discrepancy between the selected model and the calculated power spectrum of the image. Gain optimization based on the real power spectrum of the image may increase the performances of the filter bank. In this case, the optimized synthesis filter coefficients have to be sent as a side information to the decoder. It may be also interesting to study further whether subjective error criteria can be formulated as a cost function along with the subband coding gain given in (6) to obtain optimal filters.

5. CONCLUSIONS

All possible combinations of having zeros at $z = -1$ for analysis and synthesis lowpass filters for linear phase, perfect reconstruction, finite impulse response 9/7 filter bank were derived. The popular 9/7 wavelet filter bank, which has 4/4 zeros at $z = -1$, is a special case and can be derived from the gain optimized 9/7 filter bank. It was further shown that the 9/7 filter bank, which had 2/2 zeros at $z = -1$, had higher theoretical coding gain, less ringing artifact, and slightly better objective measurements than 9/7 wavelet filter bank. The maximum regularity constraint in wavelets can be relaxed and therefore other optimizing criteria may be considered.

Based on our experiments the following low-complexity filter bank model can be suggested: a moderate number of levels, but high enough to get a fairly flat passband in the lowpass band. Use 2/2 zeros at $z = -1$ with optimized

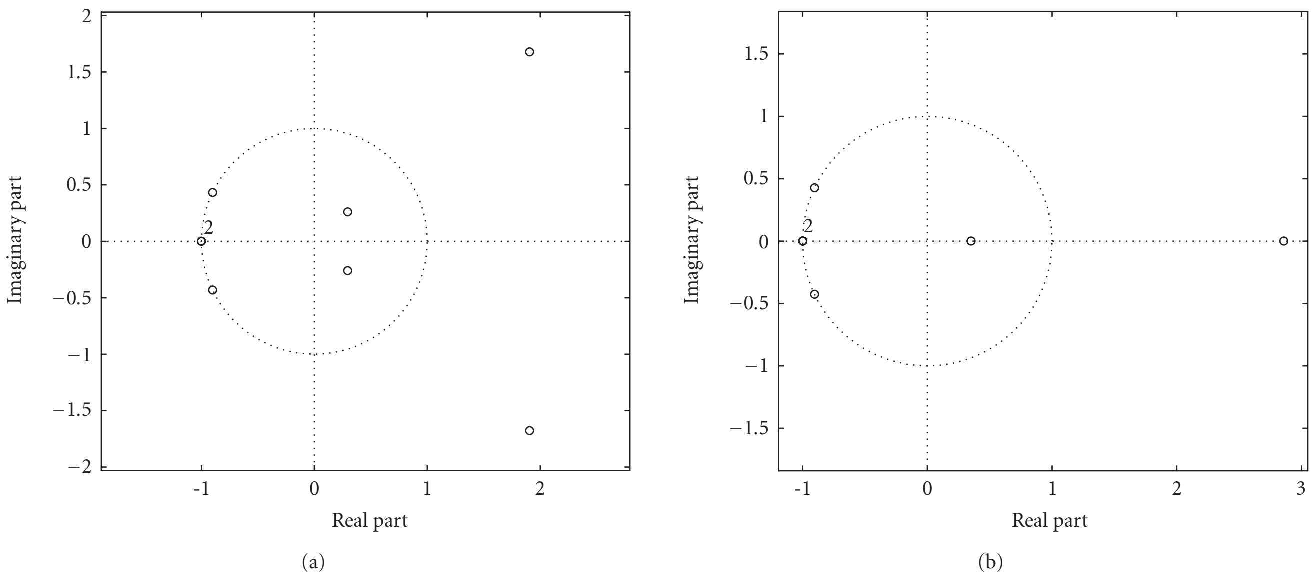


Figure 3: 2/2 zeros at $z = -1$ of the gain optimized 9/7 filter bank. (a) Analysis and (b) synthesis lowpass filters.

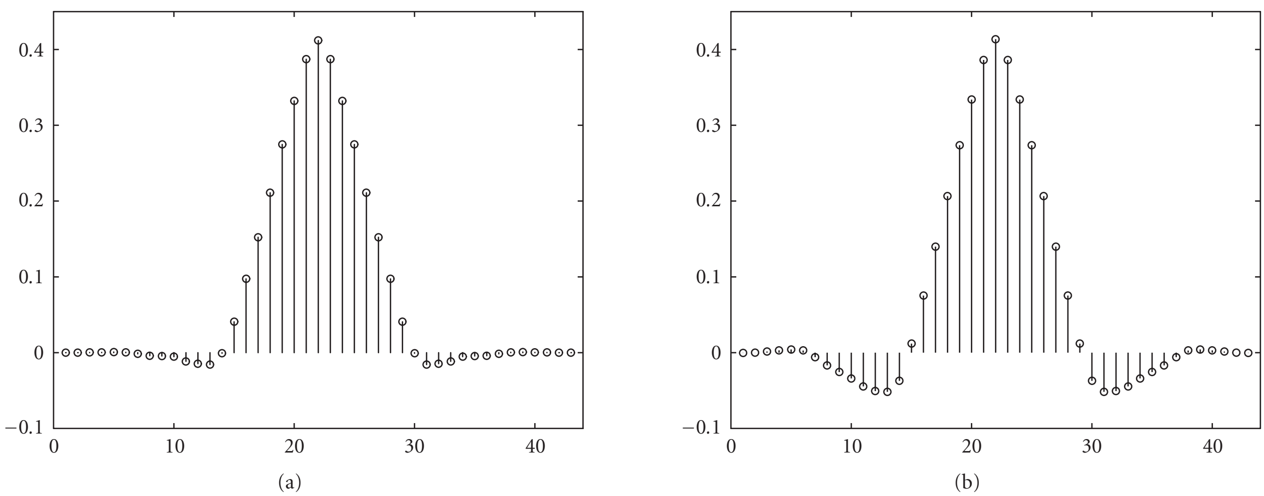


Figure 4: The 9/7 product unit sample response of the synthesis lowpass filter (43 taps). (a) Gain optimized and (b) wavelet [12].

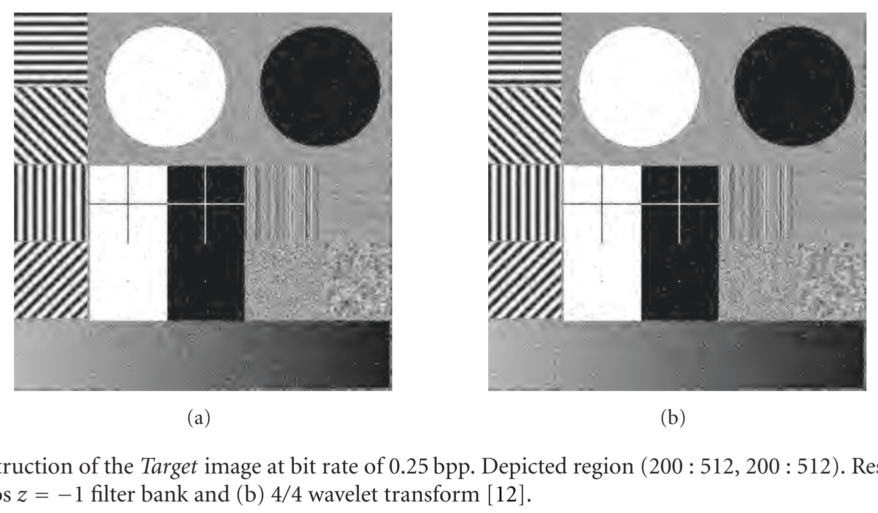


Figure 5: Lossy reconstruction of the *Target* image at bit rate of 0.25 bpp. Depicted region (200 : 512, 200 : 512). Result obtained during (a) gain optimized 2/2 zeros $z = -1$ filter bank and (b) 4/4 wavelet transform [12].

(a) (b)

Figure 6: Lossy reconstruction of the *Cafe* image at bit rate of 0.125 bpp. Depicted region (420 : 820, 100 : 400). Result obtained during (a) gain optimized 2/2 zeros $z = -1$ filter bank and (b) 4/4 wavelet transform [12].

coefficients for each image. In practice, develop a small codebook of typical filter banks from which close to optimal filters can be selected for each image. Transmit the codebook index as side information. Based on this and the bit rate, the appropriate inverse filter including Wiener filters can be derived in the receiver. This may eliminate the observed mismatch between calculated power spectra of the images and AR(1) model.

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